MUMBAI UNIVERSITY

SEMESTER -1

ENGINEERING MECHANICS QUESTION PAPER – DEC 2017

Q.1Attempt any four questions

Q.1(a) State and prove varigon's theorem.	(5 marks)

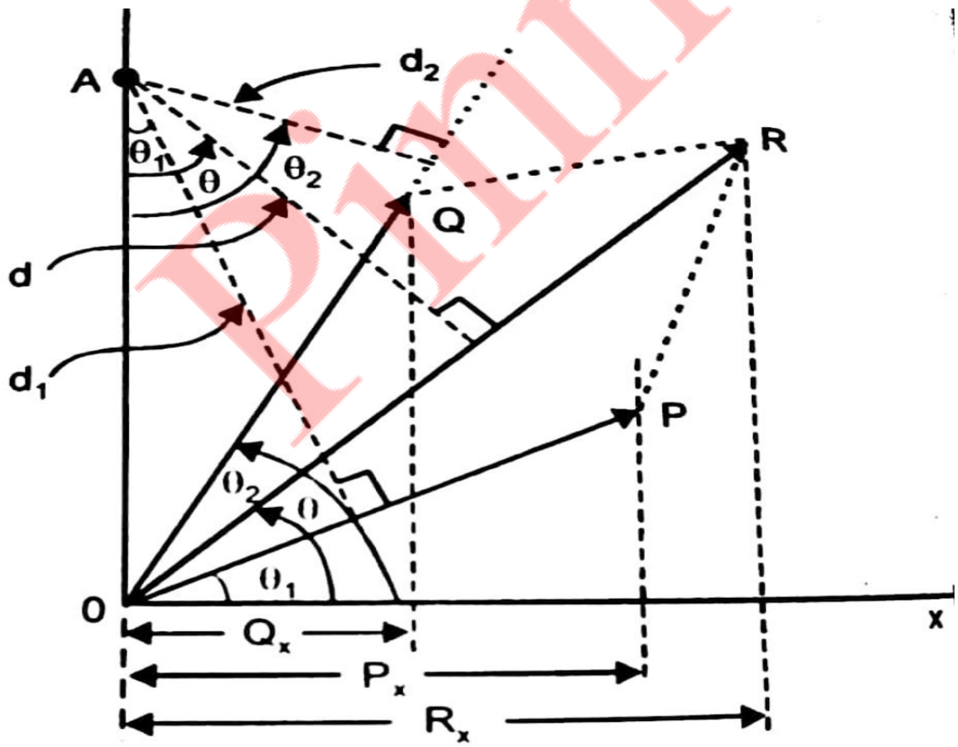
Solution:

Statement:

The algebraic sum of the moments of a system of coplanar forces about any point in the plane is equal to the moment of the resultant force of the system about the same point.

$\Sigma M_A^F = \Sigma M_A^R$

Proof:



Let P and Q be two concurrent forces at O,making angle θ_1 and θ_2 with the X-axis

Let R be the resultant making an angle θ with X axis

Let A be a point on the Y-axis about which we shall find the moments of P and Q and also of resultant R.

Let d_1,d_2 and d be the moment arm of P,Q and R from moment centre A

The x component of forces P,Q and R are P_x,Q_x and R_x

$\therefore M_A^P = P \times d_1$ ……..(1)

$\therefore M_A^Q = Q \times d_2$ ……..(2)

$\therefore M_A^R = R \times d$

$=R(\ OA.\cos\theta\ )$

$=OA.R_x$

Adding (1) and (2)

$\therefore M_A^P+M_A^Q=Pd_1+Qd_2$

$\Sigma M_A^F = P \times OA\cos\theta_1 + Q \times OA\cos\theta_2$

$=OA.P_x+OA.Q_x$ (as $P_x=P.\cos\theta_1$ and $Q_x=Q\cos\theta_2$)

$=OA(P_x+Q_x)$

$\therefore \Sigma M_A^F = OA(R_x)$ ……..(3)

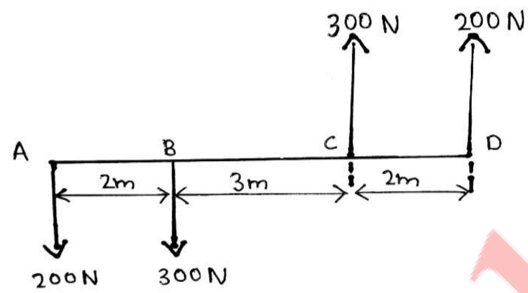
From (4) and (3)

$\Sigma M_A^F= \Sigma M_A$

Thus,Varigon's theorem is proved

Q.1(b) Find the resultant of the force system as shown in the given figure.

(5 marks)



Solution:

Taking forces having direction upwards as positive.

Net force = 200+300-200-300

=0 N

Taking moments of the forces about the point A

Taking anticlockwise moment direction as positive

$\therefore M_A = 200 \times 7 + 300 \times 5 - 300 \times 2$

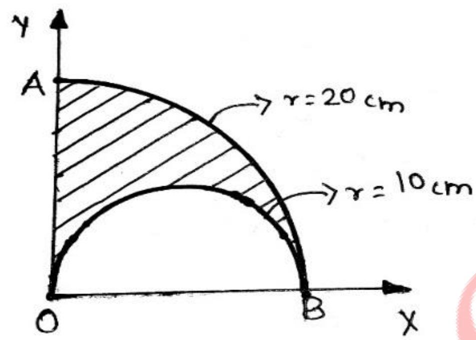
=2300 Nm (anticlockwise direction)

The resultant force is 0.

Net moment is 2300 Nm(anticlockwise)

Q.1(c) Find the co-ordinate of the centroid of the area as shown in the given figure.

(5 marks)



Solution:

Figure	Area(mm^2)	X co-ordinate of centroid (mm)	Y co-ordinate of centroid (mm)	A_x (mm^2)	A_y (mm^2)
Quarter circle	$0.25 \times \pi \times R^2$ $=0.25 \times 20^2 \times \pi$ $=314.1593$	$\frac{4R}{3\pi} = \frac{4 x 20}{3\pi}$ $=8.4883$	$\frac{4R}{3\pi} = \frac{4 x 20}{3\pi}$ $=8.4883$	2666.6667	2666.6667
Semi-circle (to be removed)	$-0.5 \times \pi \times r^2$ $= -157.0796$	10	$\frac{4R}{3\pi} = \frac{4 x 10}{3\pi}$ $=4.2441$	-1570.7963	-666.6667
Total	157.0796			1095.8704	2000

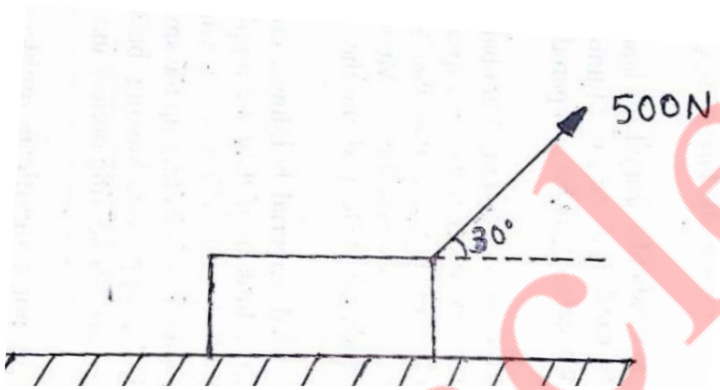
$\therefore$ X co-ordinate of centroid $(\bar{x}) = \frac{\Sigma Ax}{\Sigma A} = \frac{1095.8704}{157.0796} = 6.9765$ cm

$\therefore$ Y co-ordinate of centroid $(\bar{y}) = \frac{\Sigma Ay}{\Sigma A} = \frac{2000}{157.0796} = 12.7324$ cm

Centroid = (6.9765,12.7324) cm

Q.1(d) A force of 500 N is acting on a block of 50 kg mass resting on a horizontal surface as shown in the figure. Determine the velocity after the block has travelled a distance of 10m. Co efficient of kinetic friction is 0.5.

(5 marks)



Solution:

Given : Co-efficient of kinetic friction (μ_k)=0.5

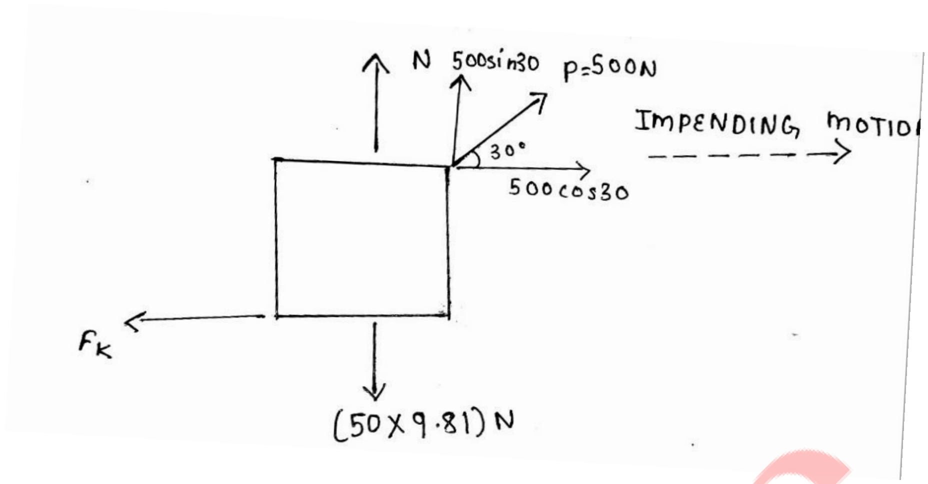
P = 500 N

m = 50 kg

u = 0 m/s

s = 10 m

To find : Velocity after the block has travelled a distance of 10 m



Solution:

The body has no motion in the vertical direction.

$\therefore \Sigma F_y = 0$

$\therefore$ N - 50g + Psin30 = 0

$\therefore$N = 50g - 500sin30

Let us assume that F is the kinetic frictional force

$\therefore$ F = μ_k x N

$\therefore$F = 0.5(50 g – 500 sin 30)

$\therefore$F = 25g – 125

By Newton's second law of motion

$\sum F_x = ma$

$\therefore$Pcos Θ – F = 50a

$\therefore$50a = 312.7627

$\therefore$ a= 6.2553 m/s^2

By kinematics equation

$v^2 = u^2 + 2 \times a \times s$

$\therefore v^2 = 0^2 + 2 \times 6.2553 \times 10$

∴ v= 11.1851 m/s

The velocity of the block after travelling a distance of 10 m = 11.1851 m/s

Q.1(e) The position vector of a particle which moves in the X-Y plane is given by

$\bar{r}$ = (3t³-4t²)$\bar{\imath}$ + (0.5t⁴)$\bar{\jmath}$ (5 marks)

Solution:

Given : $\bar{r} = (3t^3-4t^2)\bar{\imath} + (0.5t^4)\bar{\jmath}$

To find : Velocity and acceleration at t=1s

Solution:

$\bar{r} = (3t^3-4t^2)\bar{\imath} + (0.5t^4)\bar{\jmath}$

Differentiating w.r.t to t

$\therefore \frac{d\bar{r}}{dt} = \bar{v} = (9t^2-8t)\ \bar{\imath} + (2t^3)$ m/s ……..(1)

Differentiating once again w.r.t to t

$\therefore \frac{d\bar{v}}{dt} = \bar{a} = (18t-8)\ \bar{\imath} + (6t^2)\ \bar{\jmath}$

$\therefore\ \bar{a} = (18t-8)\ \bar{\imath} + (6t2)\ \bar{\jmath}$ m/s² ………..(2)

At t = 1,

Substituting t=1 in (1) and (2)

At t=1 s

$\bar{v} = \bar{\imath} + 2\bar{\jmath}$ m/s

$\bar{a} = 10\bar{\imath} + 6\bar{\jmath}$ m/s²

For magnitude :

v = $\sqrt{1^2 + 2^2}$

$=\sqrt{5}$

$=2.2361$ m/s

$a = \sqrt{10^2 + 6^2}$

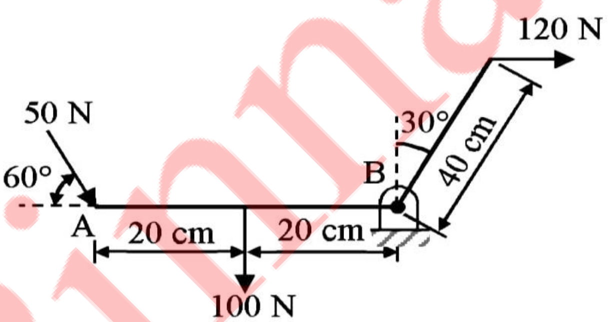
$= \sqrt{136}$

$= 11.6619$ m/s^2

Velocity at t=1s is 2.2361 m/s

Acceleration at t=1s is 11.6619 m/s^2

Q 2 a) Find the resultant of the force acting on the bell crank lever shown. Also locate its position with respect to hinge B. (8 marks)



Given : Forces on the bell crank lever

To find : Resultant and it's position w.r.t hinge B

Solution:

Let the resultant of the system of forces be R and it is inclined at an angle θ to the horizontal

The hinge is in equilibrium

Taking direction of forces towards right as positive and towards upwards as positive

Applying the conditions of equilibrium

$\Sigma F_x = 0$

$R_x = 50\cos60 + 120$

$= 145$ N

$R_y = -50\sin60 - 100$

$= -143.3013$

$R = \sqrt{R_x^2 + R_y^2}$

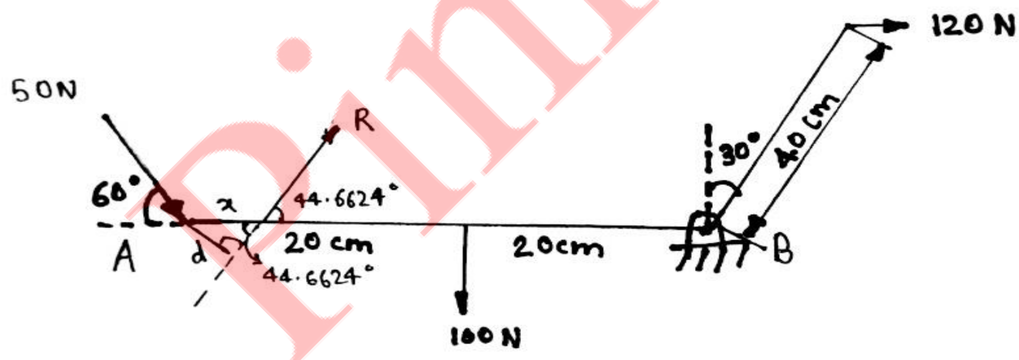
$= \sqrt{145^2 + (-143.3013)^2}$

$= 203.8633$ N

$\theta = \tan^{-1}(\frac{R_y}{R_x})$

$= \tan^{-1}(\frac{143.3013}{145})$

$= 44.6624^o$



Let the resultant force R be acting at a point x from the point A and it is at a perpendicular distance of d from point A

Taking moment of forces about point A and anticlockwise moment as positive

Applying Varigon's theorem,

203.8633 x d = -(100 x 20) – (120 x 40cos30)

d = -30.2012 cm = 30.2012 cm ………..(as distance is always positive)

sin 44.6624 = $\frac{x}{30.2012}$

x = 21.2293 cm

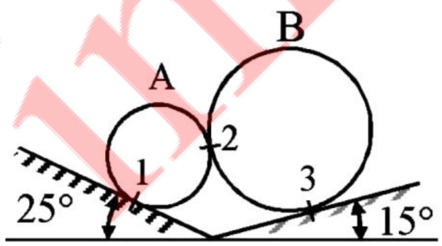
Distance from point B = 40 – 21.2293

=18.7707 cm

Resultant force = 203.8633 N (at an angle of 44.6624° in first quadrant)

Distance of resultant force from hinge B = 18.7707 cm

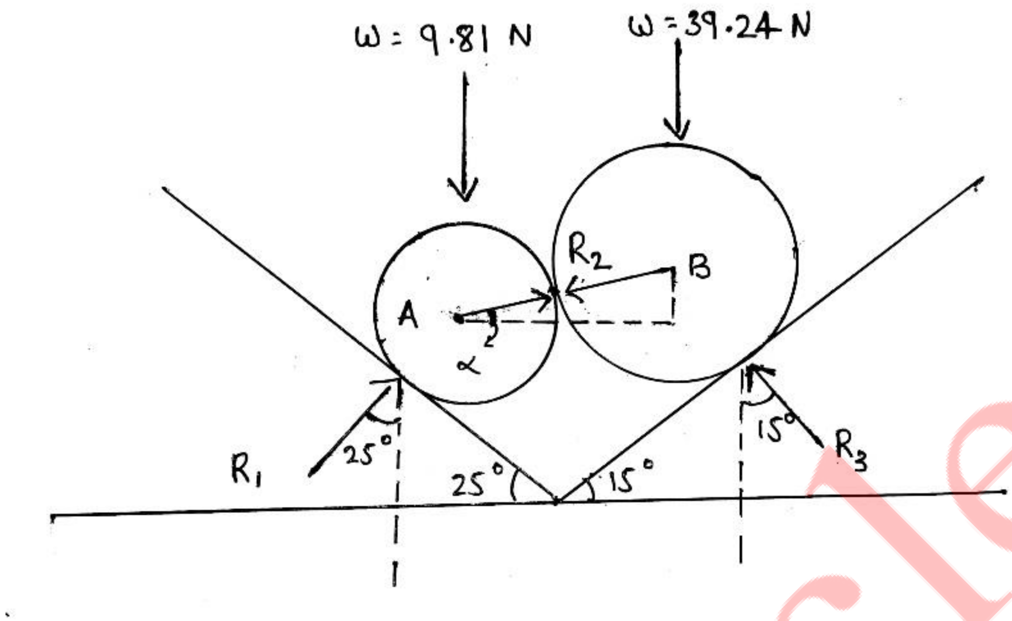
Q2b) Determine the reaction at points of constant 1,2 and 3. Assume smooth surfaces.



(6 marks)

Given: The spheres are in equilibrium

To find: Reactions at points 1,2 and 3



Solution:

Considering both the spheres as a single body

The system of two spheres is in equilibrium

Applying conditions of equilibrium:

$\sum F_y=0$

$R_1\cos25 + R_3\cos15 – g - 4g = 0$

$R_1\cos25 + R_3\cos15 = 5g$ ……(1)

$\sum F_x = 0$

$R_1\sin25 - R_3\sin15 = 0$ …….(2)

Solving (1) and (2)

$R_1 = 19.75$ N and $R_2 = 32.2493$ N …….(3)

Let the reaction force between the wo spheres be R_2 and it acts at an angle α with X-axis

Sphere A is in equilibrium

Applying conditions of equilibrium

$\sum F_y=0$

$R_1\cos25 - R_2\sin\alpha – g = 0$

$R_2 sin\alpha = 8.0896$ ……..(4) (From 3)

$\sum F_x=0$

$R_1 sin25 - R_2 cos\alpha = 0$

$R_2 cos\alpha = 19.75 sin25$

$R_2 cos\alpha = 8.3467$ ………(5)

Squaring and adding (4) and (5)

$R_2^2(cos^2\alpha+sin^2\alpha) = 135.1095$

$R_2=11.6237$ N

Dividing (4) by (5)

$$\frac{R_2 sin\alpha}{R_2 cos\alpha}=\frac{8.0896}{8.3467}$$

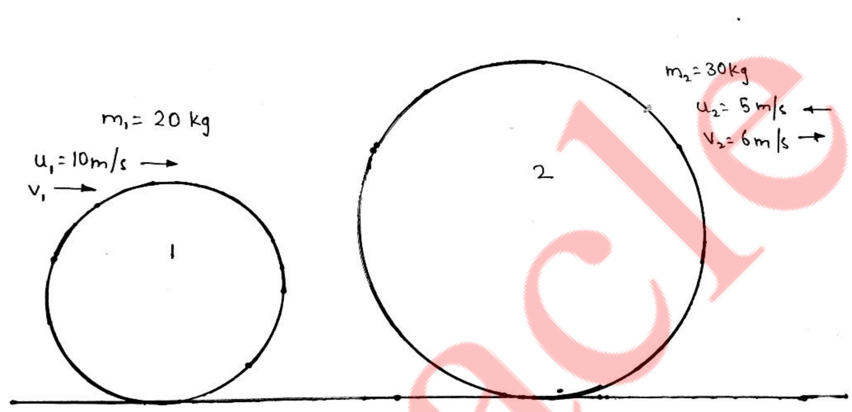
$\alpha = tan^{-1}(0.9692)$

$=44.1038^o$

$R_1=19.75$ N (75° with positive direction of X-axis in first quadrant)
$R_2=11.6237$ N (44.1038° with negative direction of X-axis in third quadrant)
$R_3=32.2493$ N (75° with negative direction of X axis in second quadrant)

Q.2 c) Two balls having 20kg and 30 kg masses are moving towards each other with velocities of 10 m/s and 5 m/s respectively as shown in the figure.

If after the impact ,the ball having 30 kg mass is moving with 6 m/s velocity to the right then determine the coefficient of restitution between the two balls. (6 marks)



Solution:

Taking direction of velocity towards right($\rightarrow$) as positive and vice versa

Given : m_1=20 kg

m_2=30 kg

Initial velocity of ball $m_1(u_1)$=10 m/s

Initial velocity of ball $m_2(u_2)$= -5 m/s

Final velocity of ball $m_2(v_2)$ = 6 m/s

To find : Co-efficient of restitution(e)

Solution:

This is a case of direct impact as the centre of mass of both balls lie along a same line.

According to the law of conservation of momentum:

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$\therefore$ 20 x 10 + 30 x (-5) =20 x v1 +30 x 6

$\therefore$ 200 – 150 =20 x v1 + 180

$\therefore$ -130 = 20 x v1

$\therefore$ v1 = -6.5 m/s

Co-efficient of restitution (e) = $(v2 - v1)/(u1 - u2)$

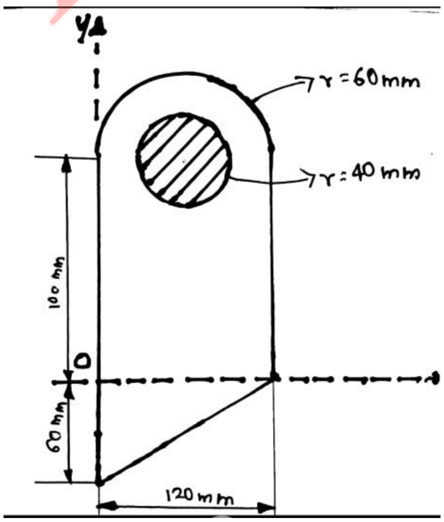
$\therefore$ e = (6 – (-6.5))/(10-(-5))

$\therefore$e=12.5/15

$\therefore$e=0.8333

The co-efficient of restitution (e) between the two balls is 0.8333

Q.3(a) Determine the position of the centroid of the plane lamina. Shaded portion is removed. (8 marks)



Solution:

FIGURE	AREA (mm^2)	X co-ordinate Of centroid (mm)	Y co-ordinate Of centroid (mm)	A_x (mm^2)	A_y (mm^2)
Rectangle	120 x 100 =12000	$\frac{120}{2} = 60$	$\frac{120}{2} = 60$	720000	600000
Triangle	$\frac{1}{2}$ x 120 x 60 =3600	$\frac{120}{3} = 40$	$\frac{-60}{3} = -20$	144000	-72000
Semicircle	$\frac{1}{2}$ x π x 60^2 =1800 π =5654.8668	$\frac{120}{2} = 60$	$100 + \frac{4*60}{3\pi}$ =125.4648	339292.01	709486.68
Circle (Removed)	- π x 40^2 =5026.5482	$\frac{120}{2} = 60$	100	-301592.89	-502654.82
Total	16228.32			901699.12	734831.86

$$\frac{\Sigma Ax}{\Sigma A} = \frac{901699.12}{16228.32} = 55.56 \text{ mm}$$

$$\frac{\Sigma Ay}{\Sigma A} = \frac{734831.86}{16228.32} = 45.28\ mm$$

Centroid is at (55.56,45.28)mm

Q3(b) Explain the conditions for equilibrium of forces in space. (6 marks)

Answer:

A body is said to be in equilibrium if the resultant force and the resultant momentum acting on a body is zero.

For a body in space to remain in equilibrium, following conditions must be satisfied:

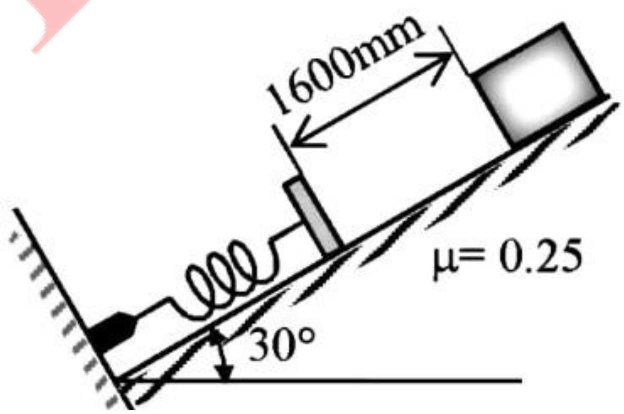
(1) Algebraic sum of the X components of all the forces is zero.
$\Sigma F_x = 0$

(2)Algebraic sum of the Y components of all the forces is zero.
$\Sigma F_y = 0$

(3)Algebraic sum of the Z components of all the forces is zero.
$\Sigma F_z = 0$

(4)Algebraic sum of the moment of all the forces about any point in the space is zero.

Q.3(c) A 30 kg block is released from rest.If it slides down from a rough incline which is having co-efficient of friction 0.25.Determine the maximum compression of the spring.Take k=1000 N/m. (6 marks)



Solution:

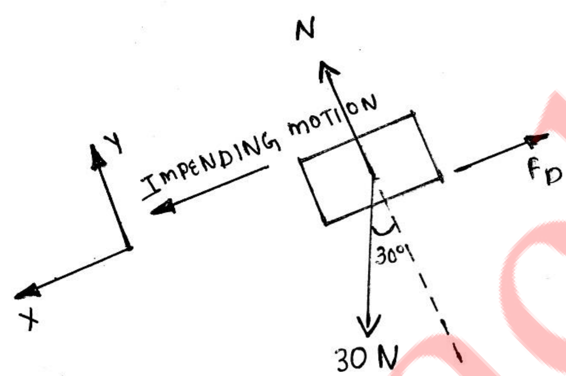
Given : Value of spring constant = 1000 N/m

W = 30N

μs = 0.25

To find : Maximum compression of the spring

Solution :



Let the spring be compressed by x cm when the box stops sliding

N = Wcos30

= 30 x 0.866

= 25.9808 N

Frictional force = μ_sN

= 0.25 x 25.9808

= 6.4952 N

Displacement of block = (1.6+x) m

Work done against frictional force = F_D x s

=6.4952(1.6+x)

At position 1

v_1=0 m/s

Vertical height above position(II) = h = (1.6+x) sin30

PE_1 = mgh = 30(1.6+x)sin30 = 15(1.6+x)

$KE_1 = \frac{1}{2}$ x mv_1^2=0

Compression of spring=0

Initial spring energy = $\frac{1}{2}$ x K x^2 = 0

At position II

Assuming this position as ground position

H^2 = 0

$P.E^2$ = 0

Speed of block v = 0

$K.E_2 = \frac{1}{2}$ x mv^2 = 0

Compression of spring = x

Final spring energy = $E_S = \frac{1}{2}$x K x (x^2)

= 0.5 x 1000 x x^2

= $500x^2$

Appling work energy principle for the position (I) and (II)

$U_{1\text{-}2} = KE_2 - KE_1$

$-W_F + PE_1 - PE_2 - E_S = KE_2 - KE_1$

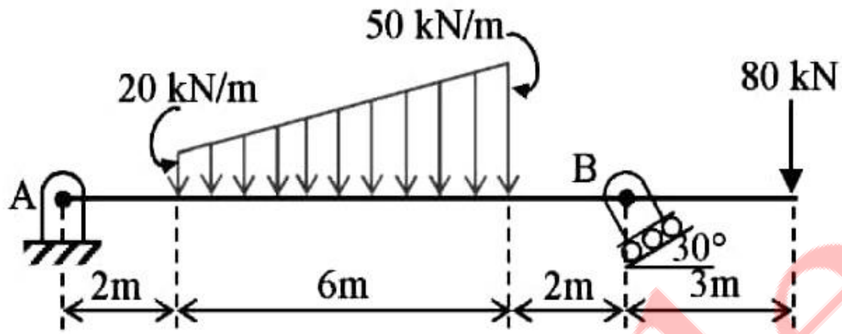
-6.4952(1.6+x) + 15(1.6+x)- 0- 500 x 2 = 0-0

$500x^2$ - 8.5048x - 13.6077= 0

x=0.1737 m

The maximum compression of the spring is 0.1737 m

Q.4(a)Find the support reactions at A and B for the beam loaded as shown in the given figure. (8 marks)



Solution:

Given : Various forces on beam

To find : Support reactions at A and B

Solution:

Draw PQ ⊥to RS

Effective force of uniform load =20 x 6 = 120 kN

$2 + \frac{6}{2} = 5$ m

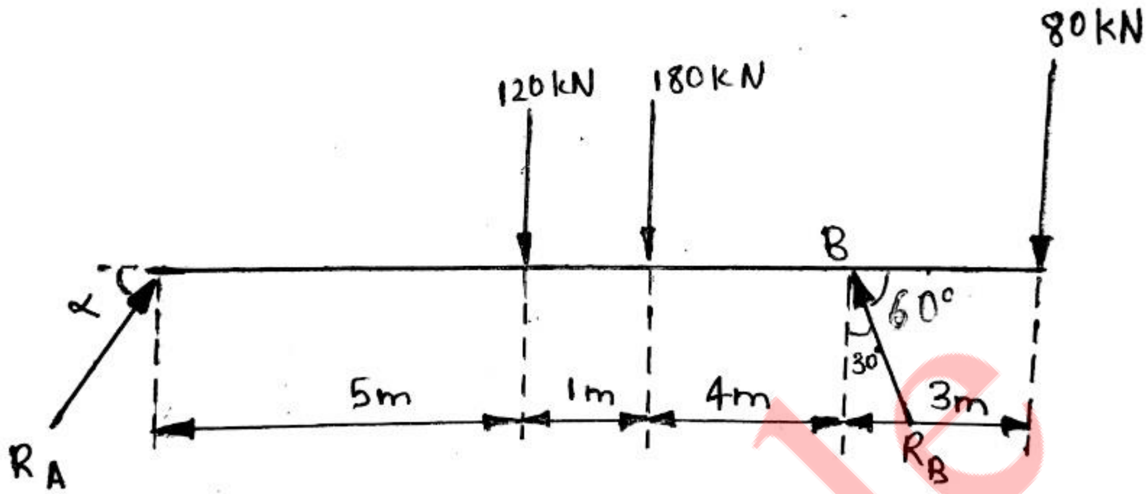
This load acts at 5m from A

Effective force of uniformly varying load =$\frac{1}{2}$ x (80-20) x 6

=180 kN

2+$\frac{6}{3}$ x 2 =6m

This load acts at 6m from A



The beam is in equilibrium

Applying the conditions of equilibrium

$\sum M_A = 0$

$-120 \times 5 - 180 \times 6 + R_B \cos 30 \times 10 - 80 \times 13 = 0$

$10 R_B \cos 30 = 120 \times 5 + 180 \times 6 + 80 \times 13$

$RB = 314.0785$ N

Reaction at B will be at 60° in second quadrant

$\sum F_x = 0$

$R_A \cos\alpha - R_B \sin 30 = 0$

$R_A \cos\alpha - 314.0785 \times 0.5 = 0$

$R_A \cos\alpha = 157.0393$ N ……..(1)

$\sum Fy = 0$

$R_A \sin\alpha - 120 - 180 + RB \cos 30 - 80 = 0$

$R_A \sin\alpha = 12 + 180 - 314.0785 \times 0.866 + 80$

$R_A \sin\alpha = 108.008$N ……..(2)

Squaring and adding (1) and (2)

$R_A^2(\sin^2\alpha + \cos^2\alpha) = 36325.3333$

$R_A = 190.5921$ N

Dividing (2) by (1)

$\frac{R_A sin\alpha}{R_A cos\alpha} = \frac{108.008}{157.0393}$

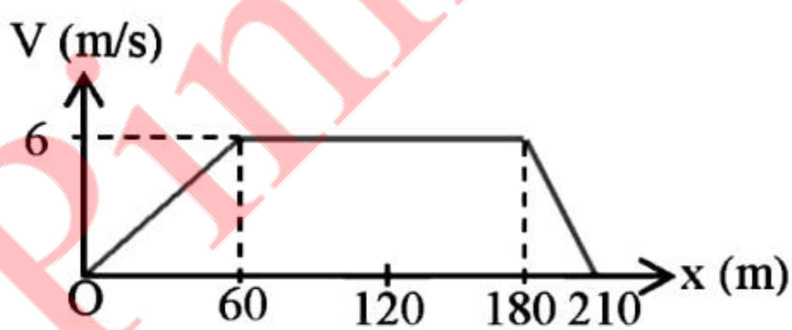
$\alpha = \tan^{-1}(0.6877)$

$= 34.5173^o$

Reaction at point A = 190.5921 N at 34.5173° in first quadrant

Reaction at B = 314.0785 N at 60° in second quadrant

Q 4b) The V-X graph of a rectilinear moving particle is shown. Find the acceleration of the particle at 20m,80 m and 200 m. (6 marks)



Solution :

Given : V-X graph of a rectilinear moving particle

To find : Acceleration of the particle at 20m,80 m and 200 m.

Solution :

$$\mathbf{a = v\frac{dv}{dx}}$$

Part 1: Motion from O to A

O is (0,0) and A is (60,6)

Slope of v-x curve $\frac{dv}{dx}=\frac{6-0}{60-0}=0.1s^{-1}$

Average velocity = $\frac{u+v}{2} = \frac{6+0}{2} = 3$ m/s

$a_{OA} = v\frac{dv}{dx} = 3 \times 0.1 = 0.3\ m/s^2$

Part 2: Motion from A to B

A is (60,6) and B is (180,6)

$\frac{dv}{dx} = \frac{6-6}{180-60} = 0\ m/s^2$

$a_{AB} = v\frac{dv}{dx} = 0\ m/s^2$

Part 3: Motion from B to C

B is (180,6) and C is (210,0)

$\frac{dv}{dx}=\frac{0-6}{210-180} = -0.2s^{-1}$

Average velocity = $\frac{u+v}{2} = \frac{6+0}{2} = 3$ m/s

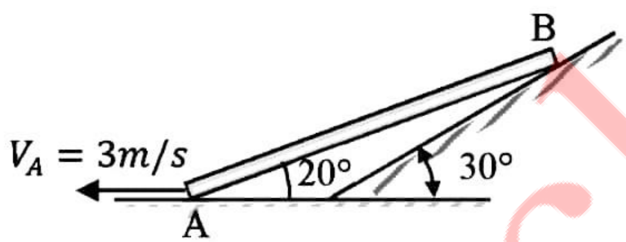
$a_{BC} = v\frac{dv}{dx} = 3 \times (-0.2) = -0.6\ m/s^2$

Acceleration of particle at x = 20 m is 0.3m/s²

Acceleration of particle at x = 80 m is 0 m/s²

Acceleration of particle at x = 200 m is -0.6 m/s²

Q.4(c) A bar 2 m long slides down the plane as shown.The end A slides on the horizontal floor with a velocity of 3 m/s.Determine the angular velocity of rod AB and the velocity of end B for the position shown. (6 marks)



Solution:

Given : $v_a = 3$ m/s

Length of bar AB = 2 m

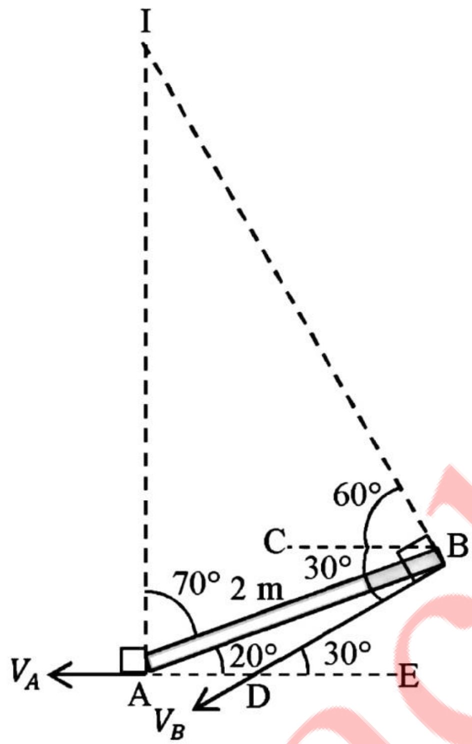
To find : Angular velocity ω

Velocity of end B

Solution:

Let ω be the angular velocity of the rod AB

ICR is shown in the free body diagram



Using Geometry:

∠BDE=30°, ∠BAD=20°

∠CBD= ∠BDE=30°

∠CBA= ∠BAD=20°

∠CBI=90°-30°=60°

∠ABI= ∠CBI + ∠CBA=60°+20°=80°

∠BAI=90°-20°=70°

In △IAB, ∠AIB=180°-80°-70°=30°

By sine rule, $\frac{AB}{\sin I} = \frac{IB}{\sin A} = \frac{IA}{\sin B}$

$\therefore \frac{2}{sin30} = \frac{IB}{\sin 70} = \frac{IA}{\sin 80}$

$\therefore$ IB = $\frac{2sin70}{\sin 30}$ = 3.7588 m

$\therefore$ IA = $\frac{2\sin 80}{\sin 30}$ = 3.9392 m

$\therefore$ Angular velocity of the rod AB = $\frac{va}{r}$ = $\frac{3}{3.9392}$ = 0.7616 rad/s (clockwise direction)

$\therefore$ Instantaneous velocity of point B = rω = IB x ω = 3.7588 x 0.7616 = 2.8626 m/s

The instantaneous velocity at point B is always inclined at 30° in the third quadrant (as shown in the free body diagram)

Angular velocity of the rod AB = 0.7616 rad/s (clockwise)

Instantaneous velocity at point B = 2.8626 m/s (30° ↙)

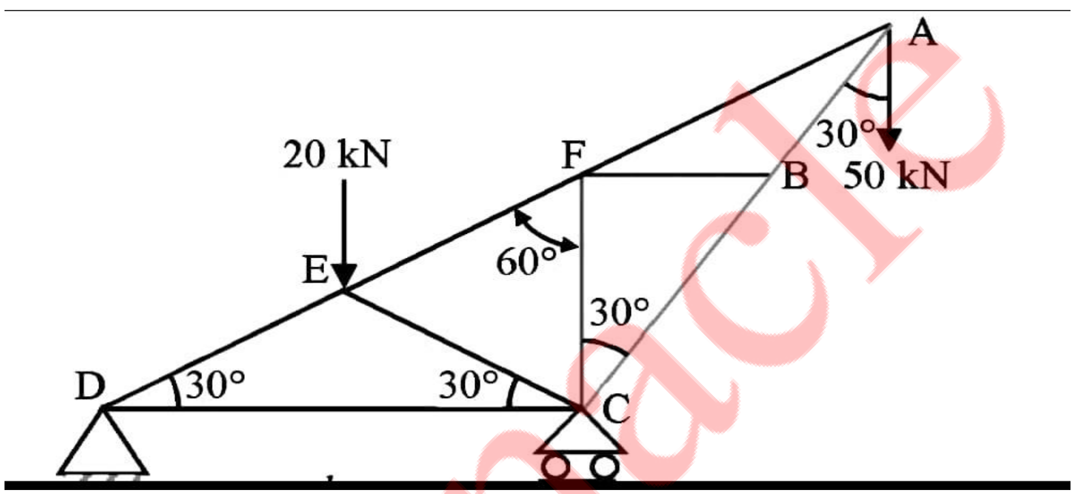
Q.5(a)Referring to the truss shown in the figure. Find :

(a) Reaction at D and C

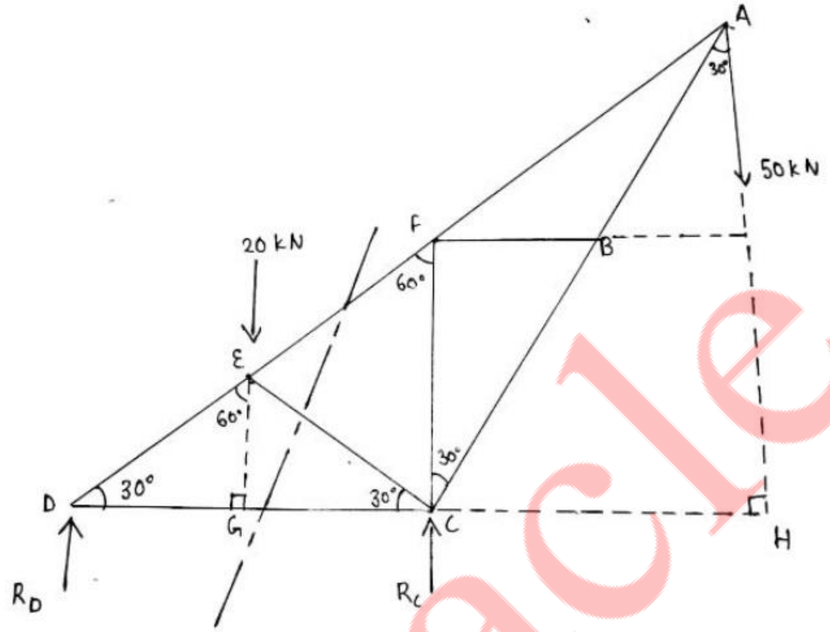
(b)Zero force members.

(c)Forces in member FE and DC by method of section.

(d)Forces in other members by method of joints. (8 marks)



Solution:



By Geometry:

In Δ ADC, ∠ADC = ∠CAD = 30°

AC = CD = l

Similarly, in Δ EDC,

ED = EC

Δ DEG and Δ CEG are congruent

DG = GC = $\frac{l}{2}$

In Δ DEG, ∠EDG=30°, ∠DGE=90°

tan30 = $\frac{EG}{DG}$

EG = DG.tan30 = $\frac{l}{2}$ x $\frac{1}{\sqrt{3}}$ = $\frac{l}{2\sqrt{3}}$

In Δ ACH,

CH = $\frac{AC}{2}$ = $\frac{l}{2}$

DH = DC + CH = l + $\frac{l}{2} = \frac{3l}{2}$

No horizontal force is acting on the truss, so no horizontal reaction will be present at point A

The truss is in equilibrium

Applying the conditions of equilibrium

Σ $M_D = 0$

-20 x DG -50 x DH + RC x DC = 0

-20 x $\frac{l}{2}$-50 x $\frac{3l}{2}$ + RC x l = 0

-10 - 75 + RC = 0

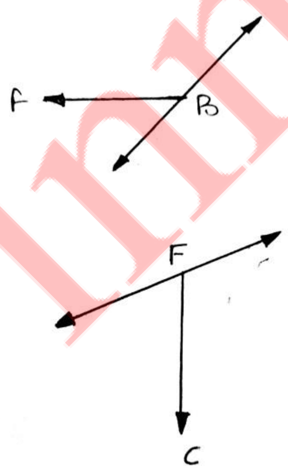
$R_C = 85$ kN

Σ F_y=0

-20 – 50 + R_D + R_C=0

R_D = -15kN

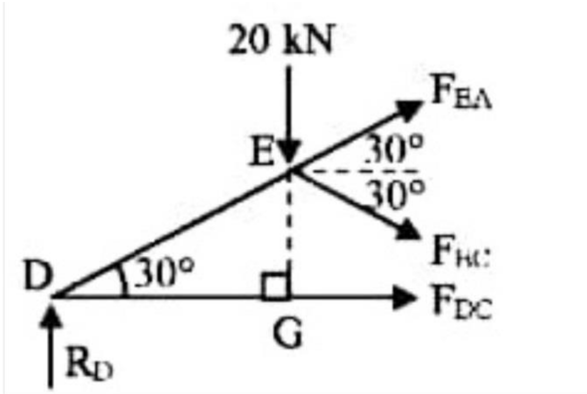
Loading at point B and F is shown



As per the rule,member BF will have zero force and is a zero force number.

Similarly,Member CF will have zero force

Method of sections :



Applying the conditions of equilibrium to the section shown

$\Sigma M_D = 0$

$-20 \times DG - F_{EC}\cos 30 \times EG - F_{EC}\sin 30 \times DG = 0$

$-20 \times \frac{l}{2} \times - F_{EC}\cos 30 \times EG - F_{EC}\sin 30 \times DG = 0$

$-20 \times \frac{l}{2} \times -F_{EC} \times \frac{\sqrt{3}}{2} \times \frac{l}{2} -F_{EC} \times \frac{1}{2} \times \frac{l}{2} = 0$

$-10 \times l - F_{EC} \times \frac{l}{4} - F_{EC} \times \frac{l}{4} = 0$

$-\frac{2l}{4} F_{EC} = 10L$

$F_{EC} = -20kN$

$R_D - 20 - F_{EC}\sin 30 + F_{EA}\sin 30 = 0$

$-15 - 20 + 20 \times 0.5 + F_{EA} \times 0.5 = 0$

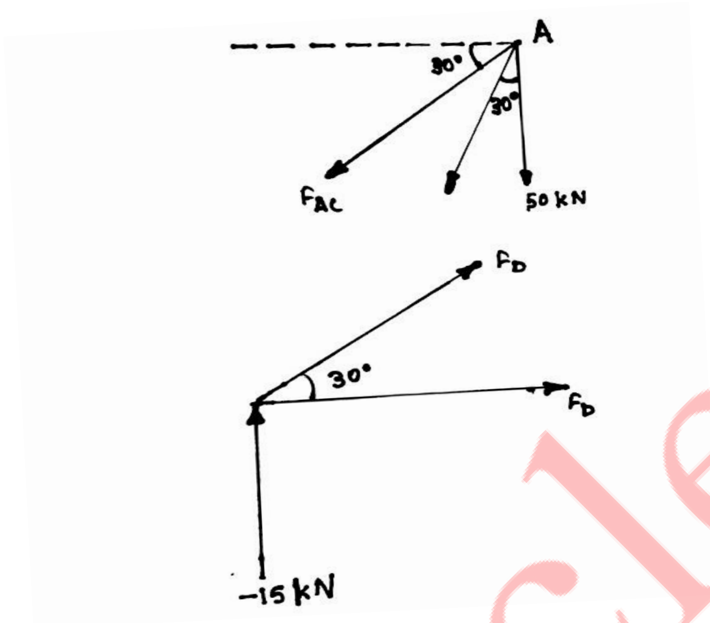
$F_{EA} = 50kN$

$F_{EC}\cos 30 + F_{EA}\cos 30 + F_{DC} = 0$

$-20 \times 0.866 + 50 \times 0.866 + F_{DC} = 0$

$F_{DC} = - 25.9808kN$

Method of joints:



Joint A

$-50 - F_{AE}\sin 30 - F_{AC}\cos 30 = 0$

$-50 - 50 \times 0.5 = F_{AC} \times 0.866$

$F_{AC} = -86.6025\text{kN}$

Joint D

$F_{DC} + F_{DE}\cos 30 = 0$

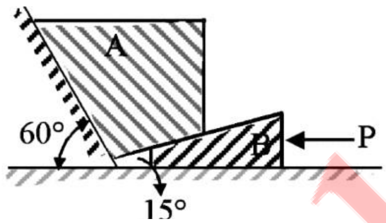
$-25.9808 + 0.866F_{DE} = 0$

$F_{DE} = 30\text{kN}$

Final answer :

Member	Magnitude (in kN)	Nature
AE (AF and EF)	50	Tension
AC (AB and BC)	86.6025	Compression
EC	20	Compression
DE	30	Tension
DC	25.9808	Compression
FB	0	
FC	0	

Q.5b) Determine the force P required to move the block A of 5000 N weight up the inclined plane, coefficient of friction between all contact surfaces is 0.25. Neglect the weight of the wedge and the wedge angle is 15 degrees. (6 marks)

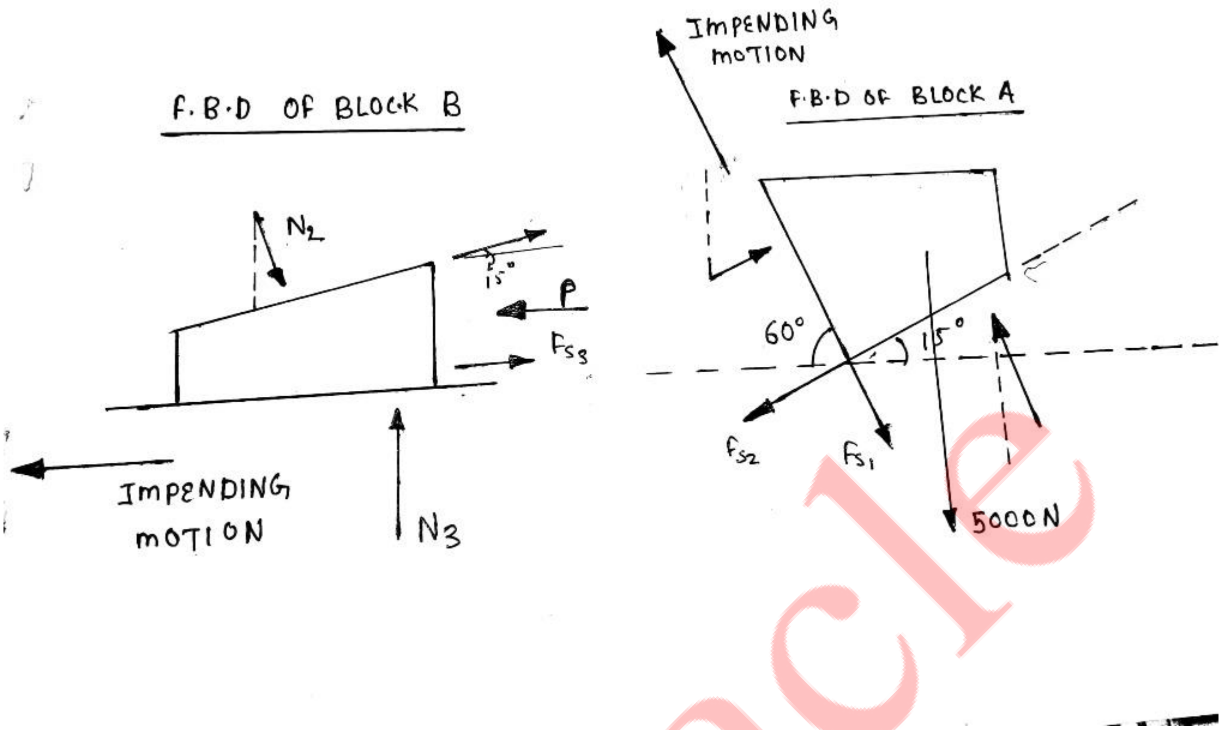


Given : Weight of block A = 5000 N

μ_s=0.25

Wedge angle = 15°

To find : Force P required to move block A up the inclined plane



Solution:

The impending motion of block A is to move up

The block A is in equilibrium

N_1,N_2,N_3 are the normal reactions

$F_{s1} = \mu_1 N_1 = 0.25N_1$

$F_{s2} = \mu_2 N_2 = 0.25N_2$

$F_{s3} = \mu_3 N_3 = 0.25N_3$

Applying the conditions of equilibrium

$\Sigma F_y = 0$

$\therefore -5000 + N_1 \cos 60 - F_{s1} \sin 60 - F_{s2} \sin 15 + N_2 \cos 15 = 0$

$\therefore N_1 \times 0.5 - 0.25N_1 \times 0.866 - 0.25 N_2 \times 0.2588 + N_2 \times 0.9659 = 5000$ (From 1)

$\therefore 0.2835 N_1 + 0.9012 N_2 = 5000$ ……………..(2)

Applying the conditions of equilibrium

$\Sigma F_x = 0$

$\therefore N_1 \sin 60 + F_{s1} \cos 60 - F_{s2} \cos 15 - N_2 \sin 15 = 0$

$\therefore 0.866\ N_1 + 0.25 \times N_1 \times 0.5 - 0.25 \times N_2 \times 0.9659 - N_2 \times 0.2588 = 0$(From 1)

$\therefore 0.991\ N_1 - 0.5003\ N_2 = 0$

Solving equation, no 2 and 3

$N_1 = 2417.0851$ N

$N_2 = 4787.79$ N

The impending motion of block B is towards left

Block B is in equilibrium. Applying the conditions of equilibrium

$\Sigma F_y = 0$

$\therefore N_3 + F_{s2} \sin 15 - N_2 \cos 15 = 0$

$\therefore N_3 + 0.25 N_2 \times 0.2588 - N_2 \times 0.9659 = 0$

$\therefore N_3 - 0.9012\ N_2 = 0$

$\therefore N_3 = 0.9012 \times 4787.79 = 4314.7563$

Applying conditions of equilibrium

$\Sigma F_x = 0$

$\therefore -P + F_{s3} + F_{s2} \cos 15 + N_2 \sin 5 = 0$

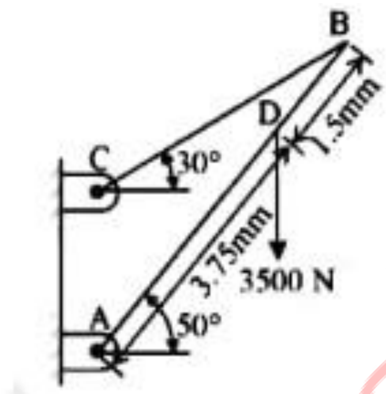
$\therefore 0.25\ N_3 + 0.25\ N_2 \times 0.9659 + N_2 \times 0.2588 = P$

$\therefore P = 0.25\ N_3 + 0.5003\ N_2 = 0.25 \text{ X } 4314.7563 + 0.5003 \times 4787.79 = 3474$ N

The force P required to move the block A of weight 5000 N up the inclined plane is P=3474 N

Q 5c) Determine the tension in a cable BC shown in fig by virtual work method.

(6 marks)



Given: F=3500 N

ϴ = 50o

Length of rod = 3.75 mm + 1.5 mm = 5.25 mm

To find : Tension in cable BC

Solution:

Let rod AB have a small virtual angular displacement θ in the clockwise direction

No virtual work will be done by the reaction force RA since it is not an active force

Assuming weight of rod to be negligible

Let A be the origin and dotted line through A be the X-axis of the system

Active force(N)	Co-ordinate of the point of action along the force	Virtual displacement
3500	Y co-ordinate of D=y_D=3.75sin θ	δy_D=3.75cos θδθ
Tcos30	X co-ordinate of B=x_B=5.25cos θ	δx_B=-5.25sin θδθ

Tsin30	Y co-ordinate of B=y_B=5.25sin θ	δy_B=5.25cos θδθ

By principle of virtual work :

-3500 x yd -Tsin 30 x yB -Tcos 30 x xB =0

-3500(3.75cos θδθ) – Tsin30(5.25cos θδθ) -Tcos30(-5.25sin θ δθ) = 0

Putting value of θ = 50° and dividing the above equation by δθ

(-3500 x 3.75cos50) – (Tsin30 x 5.25 cos50) + (Tcos30 x 5.25 sin50) = 0

5.25T(-sin30.cos50 + cos30.sin 50) = 3500 x 3.75 cos 50

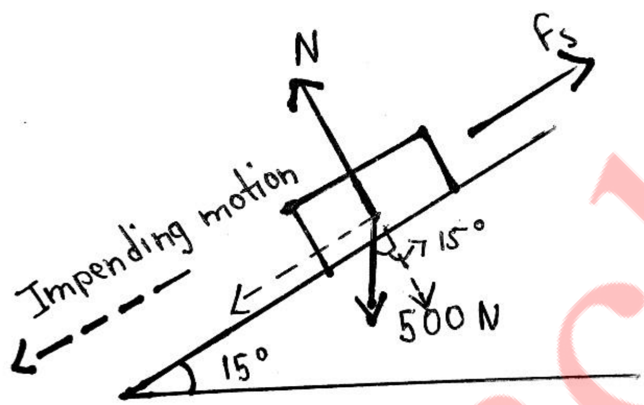
$$T = \frac{3500 * 3.75cos50}{5.25(\cos 30.sin50 - sin30.cos50}$$

$$= \frac{3500 * 3.75cos50}{5.25sin20}$$

= 4698.4631 N

The tension in the cable BC is 4698.4631 N

Q 6a) A 500 N Crate kept on the top of a 15° sloping surface is pushed down the plane with an initial velocity of 20m/s. If μs = 0.5 and μk = 0.4, determine the distance travelled by the block and the time it will take as it comes to rest. (5 marks)



Given: Weight of crate = 500 N

Initial velocity(u) = 20 m/s

μs = 0.5

μk = 0.4

θ = 15º

Final velocity (v) = 0 m/s

To find: Distance travelled by the block

Time it will take before coming to rest

Solution:

Mass (M) = $\frac{W}{g}$

$= \frac{500}{9.81}$

=50.9684 kg

Normal reaction (N) on the crate = 500 cos 15

Kinetic friction (F_k) = μ_k x N

= 0.4 x 500cos 15

= 193.1852 N

Let T be the force down the incline

Taking forces towards right of the crate as positive and forces towards left as negative

T + F_k = 500sin15

∴ T = 500sin15 – 193.1852

∴ T = -63.7756 N

By Newton's second law of motion

a = F/m

∴ a = $\frac{-63.7756}{50.9684}$ = -1.2513 m/s^2

Using kinematical equation:

$v^2 = u^2 + 2as$

∴ 0 = 202 – 2 x 1.2513 x s

∴ s = 159.8366 m

Using kinematical equation:
v = u + at

∴ 0= 20 – 1.2513t

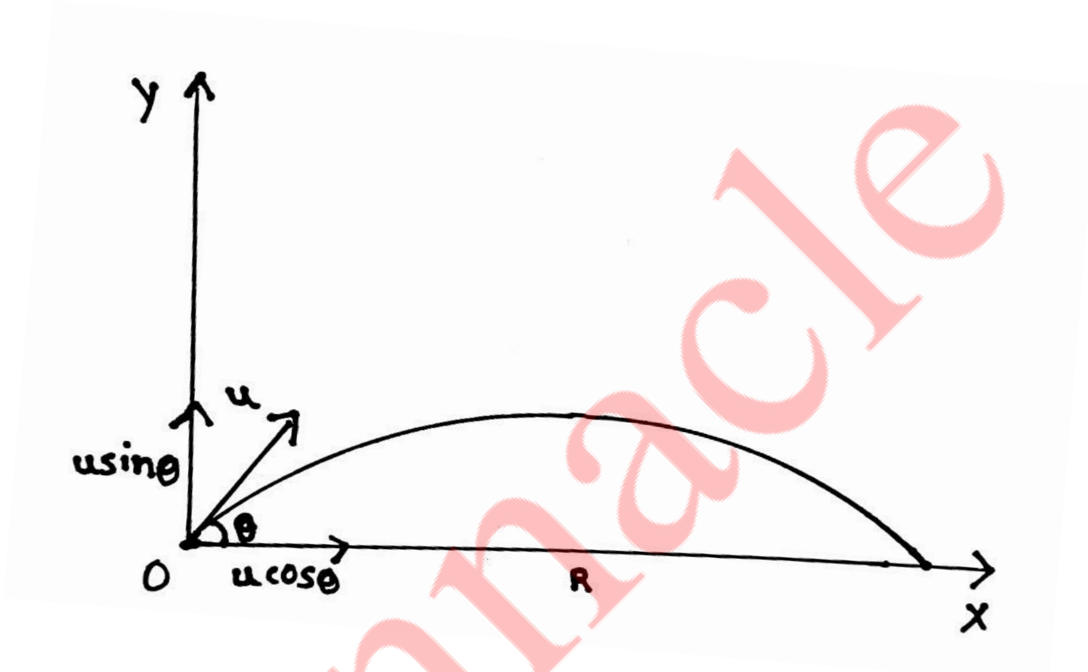
∴ t = 15.9837 s

∴ Distance travelled by the block before stopping = 159.8366 m

∴ Time taken by the block before stopping = 15.9847 s

Q.6b)Derive the equation of path of a projectile and hence show that equation of path of projectile is a parabolic curve. (5 marks)

Solution :



Let us assume that a projectile is fired with an initial velocity u at an angle θ with the horizontal.
Let t be the time of flight.
Let x be the horizontal displacement and y be the vertical displacement.

HORIZONTAL MOTION :

In the horizontal direction,the projectile moves with a constant velocity.

Horizontal component of initial velocity u is u.cosθ

Displacement = velocity x time

x = u.cosθ x t

$t = \frac{x}{ucos\theta}$

VERTICAL MOTION OF PROJECTILE:

In the vertical motion,the projectile moves under gravity and hence this is an accelerated motion.
Vertical component of initial velocity u = u.sinθ
Using kinematics equation :
$s = u_y t + \frac{1}{2} \times a \times t^2$

$y = u\sin\theta \times \frac{x}{u\cos\theta} - \frac{1}{2} \times g \times \left(\frac{x}{u\cos\theta}\right)^2$

$\mathbf{y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}}$

This is the equation of the projectile

This equation is also the equation of a parabola

Thus, proved that path traced by a projectile is a parabolic curve.

Q.6c)A particle is moving in X-Y plane and it's position is defined by
$\bar{r} = (\frac{3}{2}t^2)\bar{\imath} + (\frac{2}{3}t^3)\bar{\jmath}$ .Find radius of curvature when t=2sec. (5 marks)

Solution :

Given : $\bar{r} = (\frac{3}{2}t^2)\bar{\imath} + (\frac{2}{3}t^3)\bar{\jmath}$

To find : Radius of curvature at t = 2sec.

Solution :

Differentiating $\bar{r}$ w.r.t to t

$\frac{d\bar{r}}{dt} = \bar{v} = (\frac{3}{2} \times 2t)\bar{\imath} + (\frac{2}{3} \times 3t^2)\bar{\jmath}$

$\bar{v} = 3t\bar{\imath} + 2t^2\bar{\jmath}$

Once again differentiating w.r.t to t

$\frac{d\,\bar{v}}{dt} = \bar{a} = 3\bar{\imath} + 4t\bar{\jmath}$

$\bar{a} = 3\bar{\imath} + 4t\bar{\jmath}$

At t=2s

$\bar{v} = (3 \times 2)\,\bar{\imath} + (2\times 2^2)\,\bar{\jmath}$

$= 6\bar{\imath} + 8\bar{\jmath}$

$\bar{a} = 3\bar{\imath} + (4 \times 2)\bar{\jmath}$

$= 3\bar{\imath} + 8\bar{\jmath}$

$v = |\bar{v}| = \sqrt{6^2 + 8^2}$

$= 10$ m/s

$$\bar{a} \times \bar{v} = \begin{array}{ccc} i & j & k \\ 3 & 8 & 0 \\ 6 & 8 & 0 \end{array}$$

$= i(0-0) - j(0-0) + k(24-48)$

$= -24k$

$|\bar{a} \times \bar{v}| = 24$

Radius of curvature $= \dfrac{v^3}{|\bar{a} \times \bar{v}|} = \dfrac{10^3}{24}$

$= \mathbf{41.6667}$ **m**

Radius of curvature at t=2 s is 41.6667 m

Q.6 d) A Force of 100 N acts at a point P(-2,3,5)m has its line of action passing through Q(10,3,4)m. Calculate moment of this force about origin (0,0,0). (5 marks)

Solution :

Given: O = (0,0,0)

P= (4.5, -2)

Q= (-3,1,6)

A= (3,2,0)

F=100 N

To find : Moment of the force about origin

Solution:

Let $\bar{p}$ and $\bar{q}$ be the position vectors of points P and Q with respect to the origin O

$\therefore \overline{OP} = -2\bar{\imath} + 3\bar{\jmath} + 5\bar{k}$

$\therefore \overline{OQ} = 10\bar{\imath} + 3\bar{\jmath} + 4\bar{k}$

$\therefore \overline{PQ} = \overline{OQ} - \overline{OP} = (10\bar{\imath} + 3\bar{\jmath} + 4\bar{k}) - (-2\bar{\imath} + 3\bar{\jmath} + 5\bar{k})$

$= \mathbf{12\bar{\imath} - \bar{k}}$

$\therefore \ |PQ| = \sqrt{12^2 + (-1)^2} = \sqrt{145}$

Unit vector along PQ = $\widehat{PQ} = \frac{\overline{PQ}}{|PQ|} = \frac{\mathbf{12\bar{\imath} - \bar{k}}}{\sqrt{145}}$

Force along PQ = $\bar{F} = 100 \text{ x } \frac{\mathbf{12\bar{\imath} - \bar{k}}}{\sqrt{145}}$

Moment of F about O = $\overline{OP} \text{ x } \bar{F}$

$$= \frac{100}{\sqrt{145}} \text{ x } \begin{matrix} \bar{\imath} & \bar{\jmath} & \bar{k} \\ -2 & 3 & 5 \\ 12 & 0 & -1 \end{matrix}$$

$= 8.3045\ (-3\bar{\imath} + 58\bar{\jmath} - 36 - \bar{\boldsymbol{k}})$

$= -24.9135\bar{\imath} + 481.661\bar{\jmath} - 298.962\bar{k}$ Nm

Moment of the force = $-24.9135\bar{\imath} + 481.661\bar{\jmath} - 298.962\bar{k}$ Nm